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ABSTRACT

This is one of eighteen sets of individualized mathematics problems developed by the Oregon Vo-Tech Math Project. Each of these problem packages is organized around a mathematical topic, and contains problems related to diverse vocations. Solutions are provided for all problems. Problems presented in this package concern ratios used in food processing, gear ratios in automobiles, and electronic circuitry. The electronics problems involve complex numbers. (SD)

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ILS
INDIVIDUALIZED LEARNING SYSTEMS

Individualized Math Problems in

Algebra

Oregon Vo-Tech Mathematics Problem Sets

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SE 020 630

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Individualized Learning Systems

MATHEMATICS

Algebra

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A. Problem with Solution

1. Jam is a mixture of 45 parts fruit to 55 parts sugar and about 4 oz of pectin to a 100 lb of sugar added. This mixture is cooked until the soluble solids are 67%. What yield of jam should be produced from fruit which has 14% soluble solids? Remember: product in = product out or solids in = solids out.
Let x = lbs of water evaporated per lb of fruit entering and
y = lbs of jam formed per lb of fruit entering.

Solution:

Fruit to sugar ratio 45 to 55, therefore, one pound of fruit is mixed with 1.22 lbs sugar.

$1 + 1.22 = x + y$ $\qquad$ $0.14 + 1.22 = (0.67)y$

$2.22 = x + 2.03$ $\qquad$ $1.36 = (0.67)y$

$0.19 = x$ $\qquad$ $2.03 = y$

x = 0.19 lb of water removed per lb of fruit
y = 2.03 lbs of jam per lb of fruit

B. Problems without Solution

2. Prepare 900 gallons of a 50° Brix syrup by mixing 70° Brix syrup and 20° Brix syrup together from an unlimited supply of each of these two syrups.

B. Problems without Solution (continued)

3. How many gallons of water must be added to 500 gallons of 55° Brix syrup to form a 40° B syrup? Calculate this on a weight basis.

DEFINITIONS:

Water weighs 8.34 pounds per gallon.
The specific gravity of 55° Brix is 1.2601 and
the specific gravity of 40° Brix is 1.1787

4. A syrup room operator reduced 1500 gallons of 40° Brix syrup to a 15° Brix syrup by the addition of 500 gallons of 5° Brix syrup.

DEFINITIONS:

40° B = 8.32 x 1.1787 = 9.8304 lb per gallon
weight specific gravity
15° B = 8.34 x 1.0611 = 8.8496 lb per gallon
5° B = 8.34 x 1.0197 = 8.5043 lb per gallon

Complete Solutions to B Problems

2. x = number of gallons of 70° Brix

$900 - x$ = number of gallons of 20° Brix

$70x + (900-x)20 = (900)(50)$

$18{,}000 + 50x = 45{,}000$

$50x = 27{,}000$

$x = 540$

$900 - x = 360$ Therefore, 540 gallons of 70° Brix and 360 gallons of 20° Brix.

3. x = number of gallons of water to be added

$$(500 \text{ gal})(8.34 \text{ lbs/gal})(1.2601) = \left[(500 \text{ gal})(8.34 \text{ lbs/gal}) + (8.34 \text{ lbs/gal})x\right](1.1787)$$

$$5{,}254.6 \text{ lbs} = \left[4170 \text{ lbs} + (8.34 \text{ lbs/gal})x\right](1.1787)$$

$$5{,}254.6 \text{ lbs} = 4{,}915.2 \text{ lbs} + (9.83 \text{ lbs/gal})x$$

$$(9.83 \text{ lbs/gal})x = 339.4 \text{ lbs}$$

$$x = \frac{339.4 \text{ lbs}}{9.83 \text{ lbs/gal}} = 34.5 \text{ gal}$$

4. $40°$ B $\quad$ $5°$ B

$(1500)(9.8304) + (500)(8.5043) = (2000)(9.4989)$

should be $(2000)(8.8496)$ for $15°$ B

Therefore, there is an error of 0.6493 specific gravity or 7.34% error.

A. Problems with Solutions

1. The rear end ratio of a vehicle may not always come out to an even decimal number. A case in point is the 4.11 rear end ratio. Actually this is 4.11:1. How many teeth on the ring and pinion gears would satisfy this ratio?

   Solution:

   $10x = 41.11111.....$

   $-\ x = 4.11111.....$

   $9x = 37$

   $x = 37/9$ The ring gear would have 37 teeth. The pinion gear would have 9 teeth.

2. The pitch diameter of the cam gear is twice that of the crank gear. What is the relationship between the overall diameters of the two gears? (Let D = pitch diameter of the small gear. Let T = height of teeth above the pitch diameter circle.)

   Definitions: Pitch diameter is the diameter of the driving circle. The diameter from which gear ratio would be computed.

   Solution:

   $$\frac{2D + 2T}{D + 2T} = \frac{\text{Pitch diameter} + 2T\text{(as defined)}}{\text{Pitch diameter} + 2T\text{ (as defined)}}$$

B. Problems without Solutions

3. If the overall diameter of the crankshaft gear is 2 7/8" and the height of a tooth is 5/32" above pitch circle, what is the overall diameter of the camshaft gear? (Let D = pitch diameter of the small gear. Let T = height of teeth above the pitch diameter circle.)

B. Problems without Solutions (continued)

4. The axle gears of a given car have 14 teeth while the spider gears have 8 teeth. If, because of a low tire, the radius of one rear wheel changed from 11.5 inches to 9.5 inches and the RPM of the ring gear is 900, how many RPM do the spider gears make?

Complete Solutions to B Problems

3. $D + 2T = 2\ 7/8"$

$D + 2(5/32") = 2.875"$

$D + 0.3125" = 2.875"$

$D = 2.5625$ in

Overall diameter of camshaft gear $= 2D + 2T$

$= 2(2.5625") + 2(0.15625")$

$= 5.125" + 0.3125"$

$= 5.4375$ in

4. $2(11.5)(\pi)(900\ RPM - x) = 2(9.5)(\pi)(900\ RPM + x)$

$65{,}031.12\ RPM - 72.2568x = 53{,}721.36 RPM + 59.6904x$

$11{,}309.76\ RPM = 131.9472x$

$x = 85.71\ RPM$

$2x = 171.42\ RPM$ then

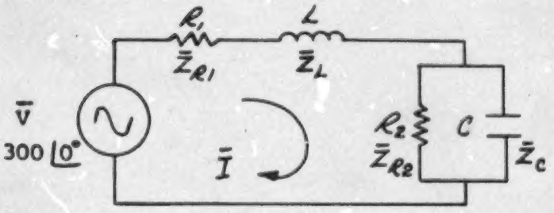
$(171.42\ RPM)(14\ teeth) = R(8\ teeth)$

$R = \frac{2399.88\ RPM\ teeth}{8\ teeth} = 300\ RPM$

The spider gears would have 300 RPM

A. Problems with Solutions

1. The R-L-C circuit below is excited by a sinusoidal generator producing a voltage phasor of, $\bar{V} = 300 \angle 0^\circ$. The impedance of each component is given in phasor form. In this case, Ohm's law applies, providing all quantities are retained in either phasor (vector) or complex number equivalent forms. Thus, $I = \frac{\bar{V}}{\bar{Z}_T}$ amps.



$$I = \frac{\bar{V}}{\bar{Z}_{R1} + \bar{Z}_L + \frac{\bar{Z}_{R2}\bar{Z}_C}{\bar{Z}_{R2} + \bar{Z}_C}}$$

$\bar{Z}_{R1} = 1.5 \angle 0^\circ$ KOhms

$\bar{Z}_L = 2.0 \angle 90^\circ$ KOhms

$\bar{Z}_{R2} = 5.2 \angle 0^\circ$ KOhms

$\bar{Z}_C = 6.0 \angle -90^\circ$ KOhms

Find the Phasor for $\bar{I}$.

Note: Leave $\bar{Z}$ in Kilohms and $\bar{I}$ will then be in milliamps.

Solution:

1. Convert all impedances to rectangular form:

$$\bar{Z}_{R1} = 1.5 \angle 0^\circ = 1.5 + J0$$

$$\bar{Z}_L = 2.0 \angle 90^\circ = 0 + J2.0$$

$$\bar{Z}_{R2} = 5.2 \angle 0^\circ = 5.2 + J0$$

$$\bar{Z}_C = 6.0 \angle -90^\circ = 0 - J6.0$$

2. Insert values and solve for I, converting from rectangular to polar forms as necessary:

$$\bar{I} = \frac{300 \angle 0^\circ}{(1.5+J0)+(0+J2.0) + \frac{(5.2 \angle 0^\circ)(6.0 \angle -90^\circ)}{(5.2+J0) + (0-J6.0)}}$$

$$= \frac{300 \angle 0^\circ}{(1.5+J2.0) + \frac{31.2 \angle -90^\circ}{7.9 \angle -49.4^\circ}}$$

$$= \frac{300 \angle 0^\circ}{(1.5+J2.0) + 3.95 \angle -40.6^\circ} = \frac{300 \angle 0^\circ}{4.54 \angle -7.6^\circ}$$

$I = 66.1 \angle 7.6^\circ$ Milliamps. Note: This result shows that 66.1 milliamps of current is flowing in the circuit with the current sinusoid leading the applied voltage sinusoid by 7.6 . The phase angle is 7.6 .

B. Problems without Solutions

2. The Phasors (Vectors) representing AC; Voltage, V; Current, I and impedance, Z, can all be represented in polar form, i.e., V = V

Voltage example: It is also useful to convert these quantities to rectangular form: V = A + JB, where A = the X-axis component, B = the Y-axis component with the operator J acting as a 90 CCW rotator. Note: The operator J in electronics is identical to the operator I in mathematics; J being used to avoid confusion with the electrical current symbol I.

Given the phasors: $\bar{V} = 100 \angle 20^\circ$ Volts; $I = 1.2 \angle -10^\circ$ Amps and $Z = 3000 \angle 43^\circ$ OHMS, express each in rectangular form.

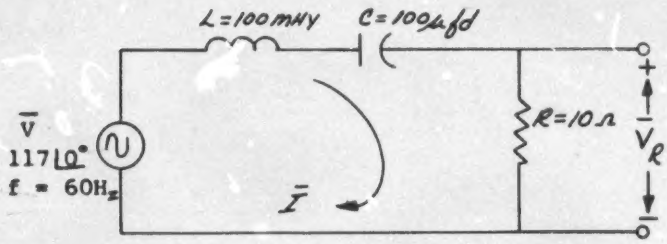
B. Problems without Solutions (continued)

3. In AC circuits, the relationships between impedance (Z), Voltage (V) and Current (I), obey Ohm's law, providing all of these quantities are expressed as Phasors. Ohm's law for AC circuits, therefore, is: $V = IZ$.

For series connected impedances, the total impedance Z_T is given by:

$\bar{Z}_T$ (Series) = $\bar{Z}_1 + \bar{Z}_2 + \ldots + \bar{Z}_N$

Given the circuit below:



$\bar{Z}_L = 2\pi fL \angle 90^\circ\ \Omega$

$\bar{Z}_C = \frac{1}{2\pi fC} \angle -90^\circ\ \Omega$

$\bar{Z}_R = R \angle 0^\circ\ \Omega$

MHY = 10^{-3} HENRYS

fd = 10^{-6} FARADS

A. Calculate $\bar{Z}_T = \bar{Z}_L + \bar{Z}_C + \bar{Z}_R$ OHMS.

B. Calculate $\bar{I} = \frac{\bar{V}}{\bar{Z}_T}$ AMPS.

C. Calculate $\bar{V}_R = \bar{I}\bar{Z}_R$ VOLTS.

Complete Solutions to B Problems

2. Voltage: $\bar{V} = 100\angle 20^\circ = 100 \cos 20^\circ + j100 \sin 20^\circ$

$\bar{V} = 94 + j34.2$ Volts

Current: $\bar{I} = 1.2\angle -10^\circ = 1.2 \cos 10^\circ - j1.2 \sin 10^\circ$

$\bar{I} = 1.18 - j0.21$ Amps

Impedance: $\bar{Z} = 3000\angle 43^\circ = 3000 \cos 43^\circ + j3000 \sin 43^\circ$

$\bar{Z} = 2194 + j2046$ OHMS

3. (a) $\bar{Z}_L = 2\pi fL\angle 90^\circ = (2\pi)(60)(100 \times 10^{-3})\angle 90^\circ = 37.7\angle 90^\circ$
$= 0 + j37.7\ \Omega$

$\bar{Z}_C = \frac{1}{2\pi fC}\angle -90^\circ = \frac{1}{(2)(60)(100 \times 10^{-6})}\angle -90^\circ$

$= 26.5\angle -90^\circ = 0 - j26.5$

$\bar{Z}_R = R\angle 0^\circ = 10\angle 0^\circ = 10 + j0$

$\bar{Z}_R = (0 + j37.7) + (0 - j26.5) + (10 + j0) = 10 + j11.2$
$= 15\angle 48.2^\circ$

(b) $\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{117\angle 0^\circ}{15\angle 48.2^\circ} = 7.8\angle -48.2^\circ$

$\bar{I} = 7.8\angle -48.2^\circ$ Amps

(c) $\bar{V}_R = \bar{I}\bar{Z}_R = 7.8\angle -48.2^\circ \times 10\angle 0^\circ = 78\angle -48.2^\circ$

$V_R = 78\angle -48.2^\circ$ Volts